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Circular Reasoning

Applying Montessori Design Patterns to Development of Software that Supports Explorations in Geometry for Elementary School Children

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It is through design that the child may be led to ponder on the geometric figures which he has handled, taken out, combined in numerous ways, and replaced. In doing this he completes an exercise necessitating much use of the reasoning facilities (Montessori, 1965, p. 301).

Using didactic materials, we have solved some problems in representation that present obstacles to the construction of the angle concept. However, other problems remain which can be addressed in the medium of computer software. For example, some angles occur in the context of rotations and revolutions, which can't be easily represented or measured with traditional representations made with paper or materials. There are also difficulties in generating precise representations of arbitrary angles that children can use for experimentation.

Later in this paper, design patterns will be applied to the design of a particular piece of software. First, however, we will consider other available software packages to see how they address the problems stated above.

Existing Educational Software

Logo

About forty years ago, a mathematician named Seymour Papert went to Geneva, Switzerland to work with the child psychologist Jean Piaget. Papert developed his own educational philosophy, a kind of constructivism that he calls *constructionism*[1]. When he came to the United States in the mid sixties, Papert worked with a team of developers at Bolt, Beranek and Newman to develop a programming language named Logo.

Since then, countless versions of the language have been developed and used in schools all over the world. Most versions of the language include a screen object called a turtle, which children can program to turn and move in various ways. The turtle can optionally

[1] Papert explains constructionism's relation to constructivism as follows:

For many educators and all cognitive psychologists, [the word "constructionism"] will evoke the term constructivism*, whose contemporary educational use is most commonly referred back to Piaget's doctrine that knowledge simply cannot be "transmitted" or "conveyed ready made" to another person. Even when you seem to be successfully transmitting information by telling it, if you could see the brain processes at work you would observe that your interlocutor is "reconstructing" a personal version of the information you think you are "conveying."* Constructionism *also has the connotation of "construction set," starting with sets in the literal sense, such as Lego, and extending to include programming languages consered as "sets" from which programs can be made, and kitchens as "sets" with which not only cakes but recipes and forms of mathematics-in-use are constructed. One of my central… tenets is that the construction that takes place "in the head" often happens especially felicitously when it is supported by construction of a more public sort "in the world"—a sand castle or a cake, a Lego house or a corporation, a computer program, a poem, or a theory of the universe* (Papert, 1993a).

leave a trail behind as it moves about the screen, and in this way the child can create any kind of geometric shape or design.

The main drawback with Logo as a tool for constructing understandings of angle is that it is difficult for students to relate turtle turns to angles as they are commonly represented. In a four year study which included a year of curriculum design and three years of testing and refinement, Clements, Battista and Sarama concluded that "compared to the traditional curriculum, [the study's Logo-based curriculum] had only moderate positive effects on the students' ability to identify and draw angles or to estimate angle size" (Clements, Battista, & Sarama, 2001).

Dynamic geometry software

Another popular type of software that supports explorations in geometry is dynamic geometry software (DGS). This includes software packages like Geometer's Sketchpad and Cabri Geometry. These packages are similar to drawing packages with embedded geometric ideas such as parallelism that allow children to set up relationships between geometric objects and observe how those relationships are maintained as they transform the objects in various ways.

Although it is targeted to middle school and older children, dynamic geometry software can potentially provide valuable experiences to children in the Montessori elementary classroom. However, studies have revealed problems with DGS as well.

Circular Reasoning

Circular Reasoning is a software package that I designed, built and introduced to a class of gifted 3rd and 4th graders at the Center for Talent Development.

The goals in developing Circular Reasoning were to (1) provide computer-based activities that are variations and extensions of existing Montessori activities, (2) solve a problem children generally have in developing a mathematical understanding (specifically, the concept of angle and related concepts), (3) provide experiences that are more effectively presented through software than other media such as didactic materials, and (4) provide this experience in the context of the two kinds of design activity—geometric design and artistic design[2].

Students need a way to relate different experiences of angle (two lines meeting at a point, a corner of a planar object, an amount of tilt or slant, rotation, revolution). CR lets them easily change sectors to angles, etc.

The Montessori classroom is an ideal setting for educational software. It allows students the time needed to learn how to use software at their own pace. Montessorians have

[2] The inspiration for this requirement is Montessori's geometric insets. In The Advanced Montessori Method II, Montessori writes:

"The designing done with these geometric insets, as will be explained, is of two kinds: geometric and artistic (mechanical and decorative). And the union of the two kinds of drawings gives new ways of applying the material… [In geometric design, the child] acquires… actual and real cognitions in geometry… [The artistic design work] facilitates the development of the child's esthetic sense (Montessori, 1965)."

experience using manipulatives to lay the groundwork for more formal understanding. However, the software must also fit the classroom.

Software for the Montessori classroom should convey a sense of familiarity to both teachers and students. This can be done through the use of screen objects that recall objects in the Montessori classroom, but more importantly through the use of feedback and activity design that match the Montessori philosophy. At least computer-based activities should be extensions and variations of existing activities. Finally, computer-based activities should provide experiences that cannot be provided more effectively through other media, such as didactic materials.

Just as all objects have three dimensions, none of which exists without the other, an angle can be thought of as having multiple aspects that occur together visually. In the traditional representation of angle, there are the lines that meet at a vertex, the area between the lines, and the angle itself, which may be thought of as the amount of rotation from one line to the next, or the fraction of a circle determined by the two sides if the vertex of the angle were at the center of the circle and the sides of the angle extended to the circumference of the circle.

Circular Reasoning is a drawing environment that provides sectors that children can manipulate in various ways (Figure 1).

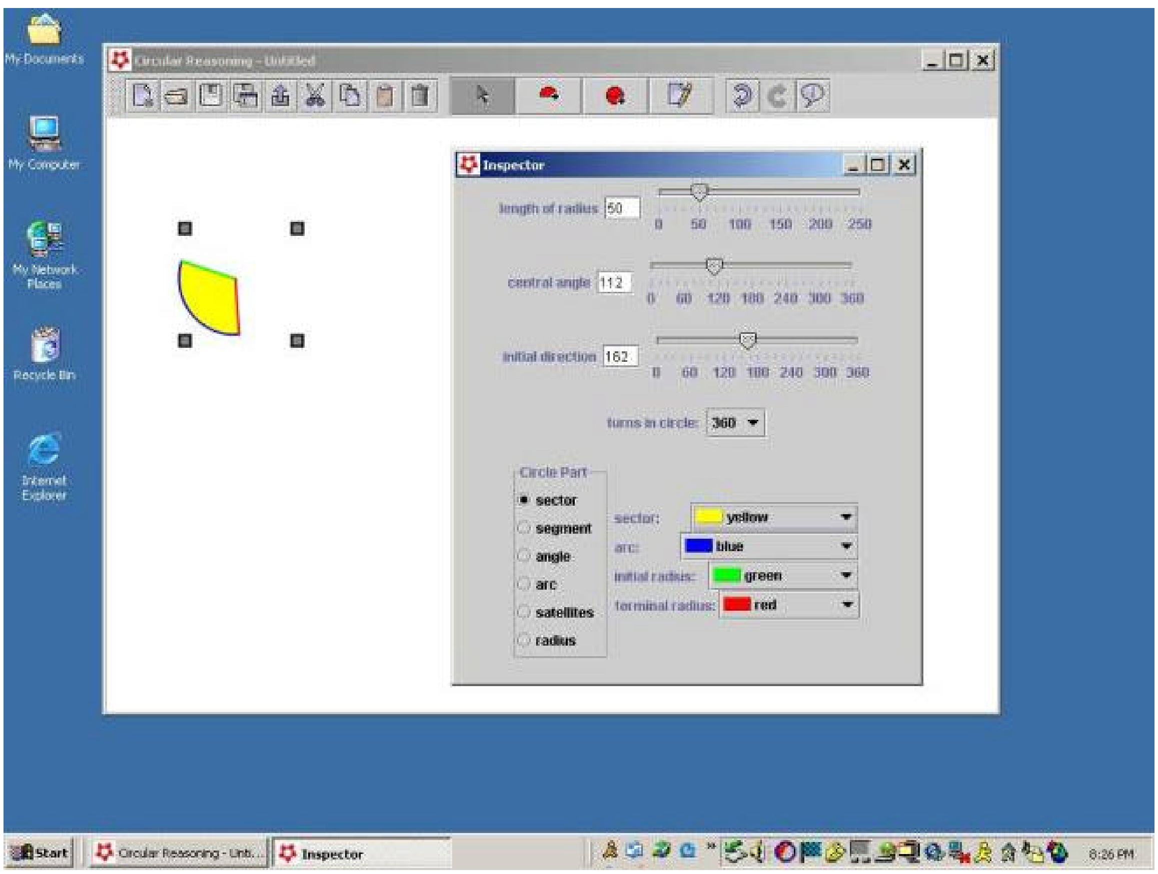

Figure 1. After a sector is placed in the workspace, the child can change its size, angle or direction, or change its representation to segment, angle, arc, satellites or radius. The various components of each representation can be separately colored.

The other primary screen objects in Circular Reasoning are the fraction circles and the free-floating notes. The fraction circles are circles divided into equal sectors. The number of sectors, representation of sectors, and relative position of sectors can be manipulated by the child. The notes are movable text boxes that children can use to make comments on their illustrations.

Design Patterns Used

In this section, I will describe the design patterns used in the development of Circular Reasoning. The last section of each design pattern describes a particular application of the pattern to the design of Circular Reasoning.

Observation before analysis

Design problem: We want to prepare a child for analysis of a form. This form has many components and aspects with which we are familiar, and we can easily highlight them or point them out to the child. However, the child has trouble differentiating these components and aspects and loses sight of their significance in relation to the whole.

Design solution: Present the form as a visual whole and give the child extensive experience with various representations of the form, providing opportunities for observation of the form under various transformations that are under the child's control. Leave room for spontaneous analysis to occur. At a later stage, if the child has not already done so spontaneously, it may be appropriate to highlight aspects or components of the form for analysis.

Example: Geometric insets are used to introduce shapes before analysis of sides and angles.[3] Children have opportunities to manipulate these shapes and thus observe them under various transformations of orientation.

Application: In Circular Reasoning, the basic components are sectors and fraction circles. After initial experience with these components and segments (the area of a circle enclosed by a chord and an arc determined by the chord), students move on to examination of arcs, angles represented as two lines meeting at a vertex, a single line with a particular "slant", and revolving "satellites".

Components of objects can be differentially colored by the child to help the child distinguish different components. The screen widgets that are used for manipulating geometric objects on the screen can also be used to measure the objects. However, the interface is designed so that children can ignore measurement until they are ready to engage in analysis.

[3] See discussions in the Montessori Method (Montessori, 1964) on pages 113, 234-235, and 243.

Known to Unknown

Design problem:	We want to introduce the child to something new. However, all learners construct new knowledge based on existing knowledge and experience. Also, an appropriate level of familiarity provides the level of comfort needed to accept the challenge of constructing new knowledge.
Design solution:	Present experiences in a sequence that begins with more familiar experiences and proceeds to less familiar ones.
Example:	Children have years of experience with the binomial and trinomial cubes before they are introduced to the cubing material.
Application:	The primary objects in Circular Reasoning are the sectors and fraction circles, which Montessori children have used in metal and plastic sets. After children have practice manipulating sectors, they proceed to work with other representations of angles. Variable Pairing, as described in the previous Design Patterns section, is also employed in Circular Reasoning. This occurs when the child changes a sector to another type of object (segment, arc, etc.) or creates a sector and pairs it with another object that is identical in angle but different in orientation, or identical in angle and orientation but different in type. This pairing supports the progression from familiar sectors (from fraction circles) to less familiar representations of angle.

Indirect Preparation

Design problem:	We want to introduce the child to a new task. However, even with repetition, the child is unable to complete the task.
Design solution:	Present related but more accessible experiences to serve as stepping stones to the more difficult task[5].
Example:	Tracing the geometric insets is indirect preparation for writing.
Application:	Circular Reasoning may be considered indirect preparation for more advanced work with Logo and dynamic geometry software. As preparation for Logo, it helps the child learn to estimate angles and distances and to recognize the Total Turtle Trip Theorem[4]. Further, it prepares the child to connect Logo geometry with Euclidean geometry through rich opportunities to associate angle-by-rotation with other representations of angles. As a preparation for dynamic geometry software, it gives the child experience with manipulating and measuring onscreen geometric objects.

[4] "If a Turtle takes a trip around the boundary of any area and ends up in the state in which it started, then the sum of all turns will be 360 degrees (Papert, 1993b)." This relation is seen in the closed polygons produced with the slide operation.

Results

Rather than focusing on specific outcomes, Montessori was more concerned with the power of an exercise to capture the child's attention and imagination. Researchers have recently begun analysis of Montessori activity in terms of flow theory (Kahn, 2003). At the time when I taught this class, it did not occur to me to attempt to apply flow theory to evaluate student performance. The results I have to share at this time are of student work and student performance on pre- and post-tests. My recollection is that the children were generally highly engaged in their work. In future investigations, it would be fruitful to collect data describing children's work in terms of flow theory.

Children's Work

In the spring of 2003, I held fifteen hours of geometry classes as a teacher for Northwestern University's Center for Talent Development. The course was called "Thinking About Circles." Classes were given over a period six weeks to a group of gifted 3rd and 4th grade children.

Children were given exercises to develop vocabulary (Figure 2) and recognize relationships between geometric objects (Figure 3).

Future Directions

It would be interesting to try to evaluate work with Circular Reasoning in terms of flow theory. The software itself could be extended in many ways to support explorations in geometry for children aged six to twelve. I have also applied Montessori design patterns to the development of some short online exercises which I have used with a gifted K-3 class (http://leonelearningsystems.com/for_students.htm).

Circular Reasoning is available for free download at http://leonelearningsystems.com/prod01.htm, where teachers and researchers can also find pdf files with which they can print out activity cards.

If you make use of Circular Reasoning or the web activities, please send any comments or questions to tj@leonelearningsystems.com.

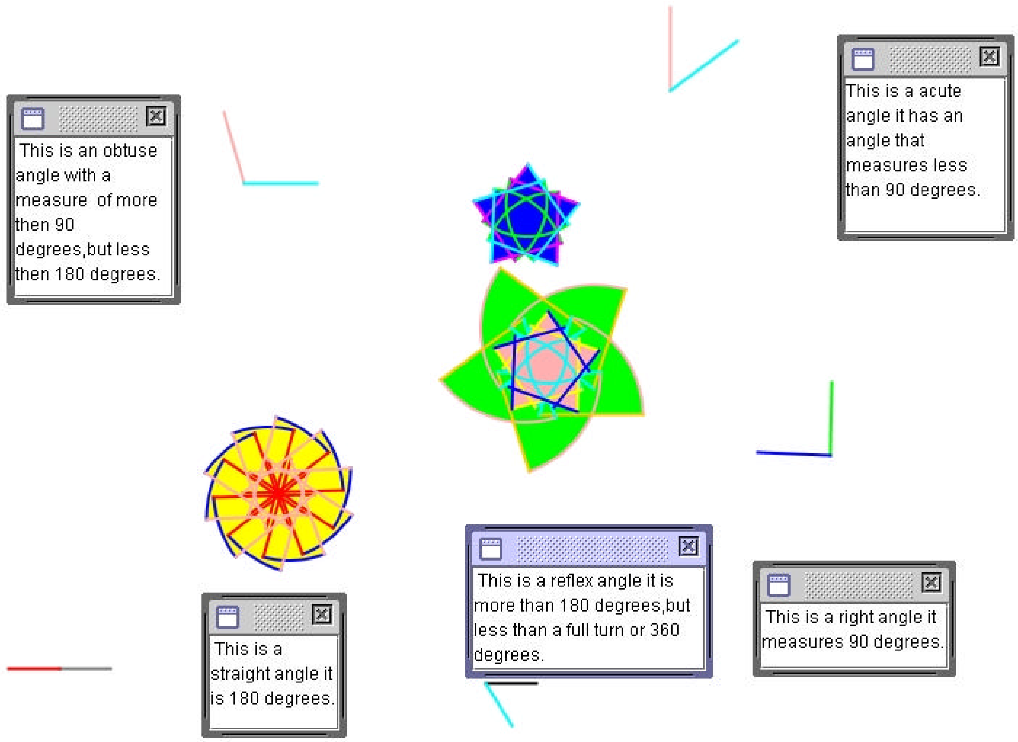

Figure 2. Sample of child's work. Task was to create illustrated definitions of various geometric objects.

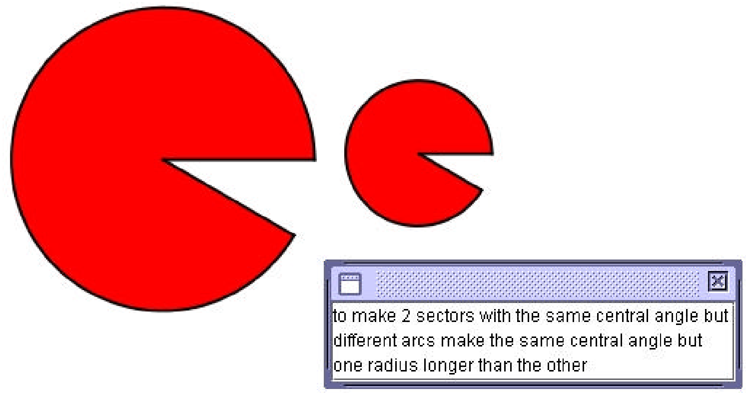

Figure 3. Sample of child's work. Puzzle posed to children was: Can you make two sectors with the same central angle but differently sized arcs?

Children also had ample opportunity to do representational (Figure 4) and abstract (Figure 5) art work with Circular Reasoning.

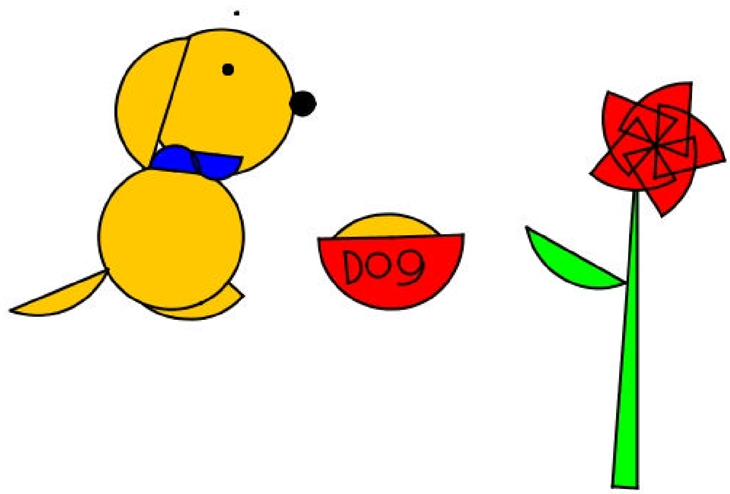

Figure 4. Sample of student's work. Flower and puppy made from sectors, segments, fraction circle and circle centers (points).

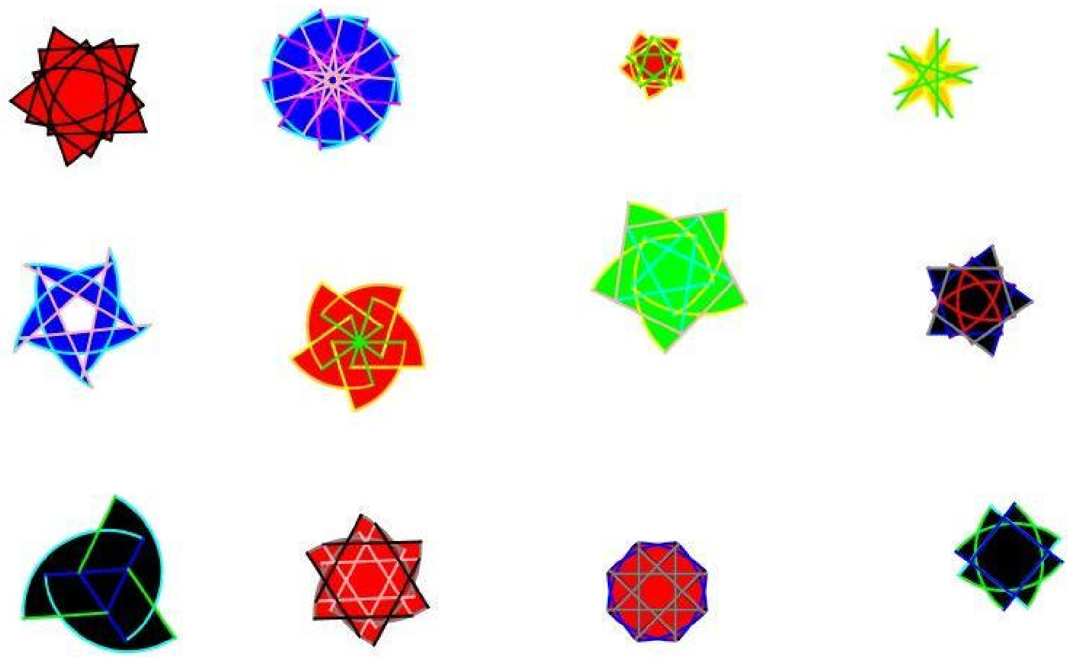

Figure 5. Sample of child's work. Geometric designs created by manipulating fraction circles.

Test Results

The Thinking About Circles class was given pre-tests and post-tests using selected questions in geometry (two open-ended and 7 multiple choice) from the National Assessment of Educational Progress (*NAEP Questions*, 2003) that were administered nationally to 8th and 12th graders.

On the pre-test, as we would expect, their performance was well below the performance of the 8th and 12th graders (*NAEP Questions*, 2003). The TAC kids had an average of 29% on the multiple-choice questions. Fourteen students took the pre-test. Seven of them got only one question right on the pre-test.

On the post-test, the TAC kids scored an average of 56% on the multiple choice questions, outperforming 8th graders on all questions administered to 8th graders except one, usually by a wide margin (Table 1).

Table 1. Comparative scores of Thinking About Circles class with national scores for 8th and 12th graders on selected NAEP geometry questions. Two scores are given for the Thinking About Circles class. "CR Pretest" are scores obtained before start of class. "CR Posttest" are scores after approximately twelve hours of discussion and work with Circular Reasoning software.

Comparative Scores

Test Question	**3**	**4**	**5**	**6**	**7**	**8**	**9**
CR Pretest							
Correct	31%	8%	15%	69%	31%	23%	31%
Incorrect	62%	54%	85%	23%	54%	54%	46%
Omitted Item	8%	38%	0%	8%	15%	31%	23%
8th Grade Scores							
Correct	32%	23%	33%	74%	-	-	-
Incorrect	67%	73%	66%	25%	-	-	-
Omitted Item	10%	4%	1%	1%	-	-	-
12th Grade Scores							
Correct	-	44%	-	-	70%	49%	79%
Incorrect	-	53%	-	-	29%	49%	20%
Omitted Item	-	3%	-	-	1%	2%	1%
CR Posttest							
Correct	46%	46%	31%	92%	46%	54%	77%
Incorrect	54%	23%	69%	8%	36%	46%	23%
Omitted Item	0%	31%	0%	0%	15%	0%	0%

Four of the questions on the test were administered nationally to 12th graders. Of those four questions, the TAC kids outperformed 12th graders on two of the questions (*NAEP Questions*, 2003).

The TAC class had one 4th grader and all the rest were 3rd graders and about 12 hours between tests for work with Circular Reasoning and classroom discussion (15 hours in attendance minus time for snacks and tests).

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The Author

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