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Colored Sectors

Applying a Montessori Design Pattern to the Problem of Angle Representation

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In this paper, I will apply a design pattern found in Montessori materials to design a new material to try to solve a particular design problem—devising a representation of angles that support the development of the angle concept in young children.

In constructing the angle concept, children often confound angle with length or area (Prescott, Mitchelmore, & White, 2002). If we look at the typical, two-dimensional representation of angle (Figure 1), it is easy to see why.

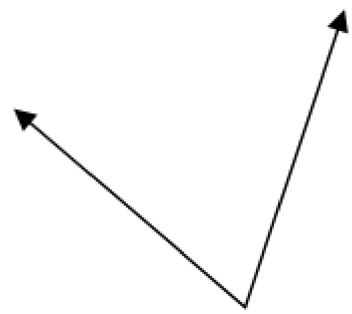


Figure 1. An angle is typically represented by two straight lines that meet at the vertex on one end and have arrows on the other end that point away from the vertex.

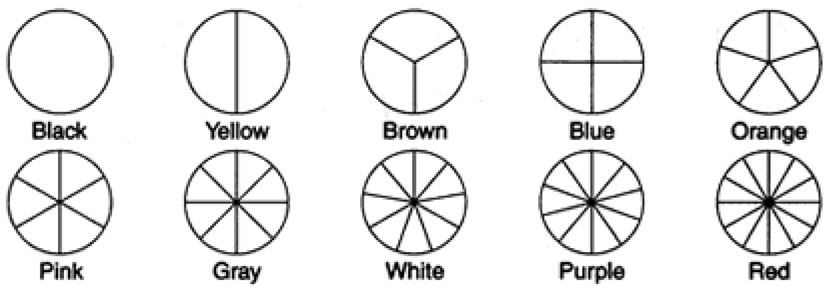
Where exactly is the angle in Figure 1? We can try to explain to the child that the angle is the space between the two lines. But if we extend the lines, there will be more space between them. So the child might wonder if that will make the angle bigger. Of course, we must answer "No", but how do we explain this?

Montessori strove to help children construct understandings through related and repeated experiences that can span several years. A progression of Montessori experiences related to angles include the presentation of various polygons, talking to a child as she sets a table with reference to the sides and corners of the table (Montessori, 1964), and fraction circles (Montessori, 1965). Other activities were later developed by Mario Montessori to support more analytical work by older children. Altogether, these activities span pre-school and elementary school years.

In this article, I will suggest a Montessori-inspired material for pre-school children that is especially designed to highlight the angle attribute of shapes. The material is called "colored sectors", and is based on Montessori's fraction circles and knobless cylinders.

Fraction Circles and Angles

In a recent report on extensive research conducted by the Rational Number Project (RNP), Kathleen Cramer and Apryl Henry presented the finding that "most children need to use concrete models over extended periods of time to develop mental images needed to think conceptually about fractions" (Cramer, 2002). They also found that "the fraction circle model used in combination with RNP activities was the most powerful of the models" (Cramer, 2002). The fraction circles recommended in this 2002 report are essentially the same ones used by Maria Montessori around the turn of the last century (Figure 2).



Fraction circles can be easily constructed by duplicating a pattern on construction paper and then cutting apart.

Figure 2. Fraction circles recommended in the NCTM 2002 Yearbook: Making Sense of Fractions, Ratios and Proportions. In Montessori's original design, all fraction circles were red. They were cut from metal, had knobs for gripping and fit into green frames. Montessori supply houses also sell all-red plastic sets (no knobs) with multiple sets of fraction pieces.

However, Montessori did not intend that fraction circles should be used only to teach about operations on fractions. Montessori classrooms also include special devices for measuring the sectors that make up the fraction circles. One of these has 100 tick marks for converting fractions into decimals (Montessori, 1965). The other is a protractor divided into degrees to measure the central angle of each sector (Montessori, 1965).

The combination of sectors and specially designed protractor is a definite improvement over the traditional representation of an angle with a traditional protractor, in the sense that it helps students to produce accurate measurements more reliably.

However, we are still confronted with the representation problem we had earlier. How do we help the child distinguish angle from other attributes of a sector? The radii of all sectors are equal in the fraction circle model, but area and arc length vary with angle. So how do we make it clear to beginning students that they are measuring angle rather than area or arc length?

How would Montessori have approached this problem? One clue lies in the design patterns we can find in existing Montessori materials.

A Design Pattern from the Knobless Cylinders

Design patterns are paired design problems and solutions. Patterns are related in a hierarchy that is referred to as a pattern language. Design patterns were first used in architecture (Alexander, Ishikawa, & Silverstein, 1977) and were later applied to software and interaction design.

Design patterns can be found in Montessori classrooms in many areas at many levels, including particular materials, classroom layout, classroom norms of behavior, school architecture, school policy—anything in or related to the classroom that can be created by humans.

To try to solve the angle-representation problem, I used a design pattern that I found in the knobless cylinders, Montessori materials that help pre-school children distinguish between thickness and height (Figure 3).

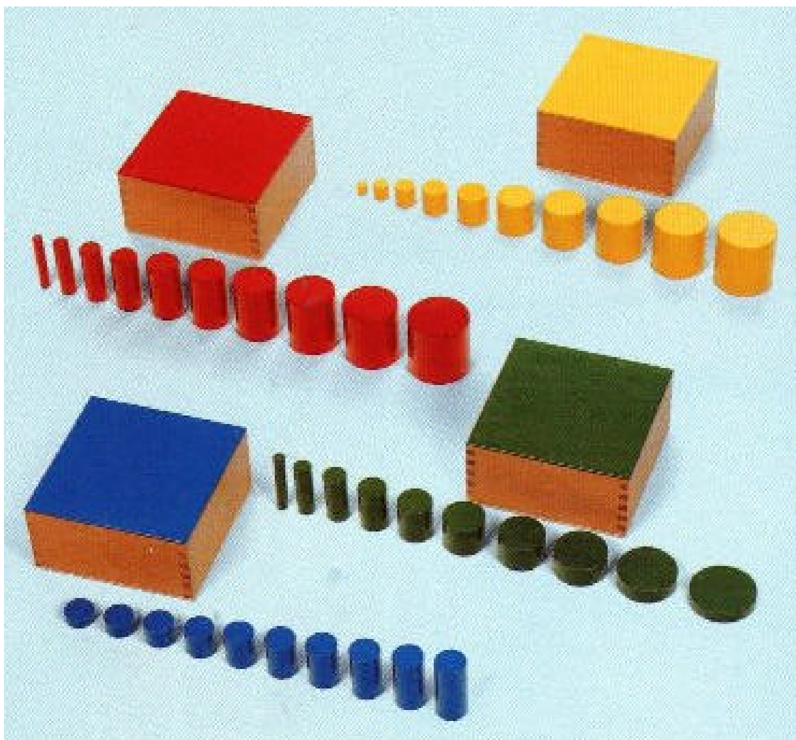


Figure 3. These cylinders were designed to help pre-school children distinguish thickness from height. The yellow cylinders get taller as they get thicker. The red cylinders increase in diameter but don't change in height. The blue cylinders increase in height without changing in diameter. The green cylinders increase in diameter as they decrease in height.

It is only after extensive experience in ordering and comparing cylinder sets (and other materials) that children are expected to understand measurements of various dimensions.

I extracted a particular design pattern from the knobless cylinders (there are others) that I call "Variable Sequencing". A description of the pattern follows.

Variable Sequencing

Design problem: We want to help the child recognize and construct understandings about attribute X. However, pairing objects will not highlight attribute X because we cannot create an object with attribute X that does not also have attribute Y, so children will be apt to confound attributes X and Y.

Design solution: If attributes X and Y can both be graded, make four sets of objects that are graded as follows:

Set 1: Objects can be graded in increasing order by both attribute X and Y.

Set 2: Objects can be graded in increasing order by attribute X. Attribute Y remains constant.

Set 3: Objects can be graded in increasing order by attribute Y. Attribute X remains constant.

Set 4: Objects can be graded in increasing order by attribute X and, simultaneously, in decreasing order by attribute Y.

Example: Knobless cylinders. In this case, attribute X is diameter and attribute Y is height. Set 1 corresponds to the yellow cylinders, set 2 to the red cylinders, set 3 to the blue cylinders, and set 4 to the green cylinders.

Colored Sectors

The Variable Sequencing pattern can be applied to sectors found in the fraction circles. We want to help the child distinguish the central angle of a sector from the size of the circle that defines the sector (the radius length or area of the enclosing circle). To apply the design pattern, we can let attribute X be angle and attribute Y be length of radius (Figure 4). This arrangement requires that students distinguish between angle and radius length in the process of sequencing the sectors in a set and comparing sectors between sets.

What Now?

This article is about a design pattern for a manipulative. However, manipulatives are little help if their use is poorly supported. Further design patterns relevant to colored sectors can be found in presentations of Montessori materials, especially the knobless cylinders.

For example, Montessori design patterns suggest that the yellow set of sectors should be introduced first, and that children should have experience with individual sets of sectors before mixing sets. The problem of how to present Variable Sequencing materials and the various presentations commonly used in Montessori classrooms could make up another design pattern.

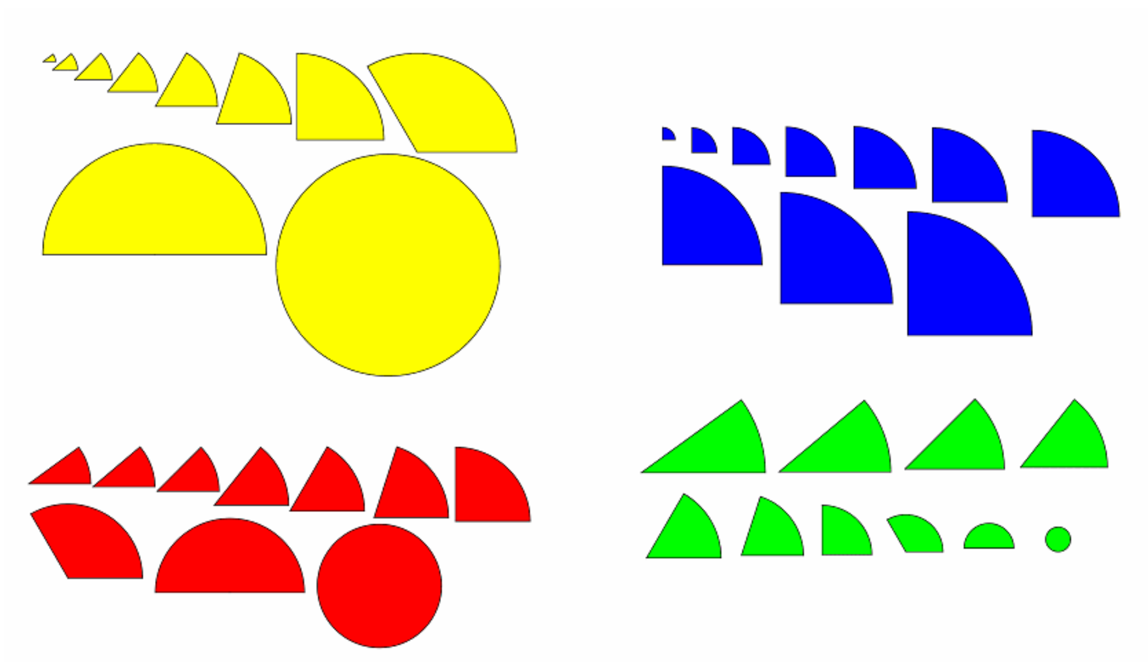


Figure 4. The yellow sectors can be placed in increasing order by both radius length and angle. The red sectors can be ordered by angle. The blue sectors can be ordered by radius length. The green sectors can be placed in increasing order by angle and simultaneously in decreasing order by radius.

Other areas to search for relevant design patterns include such artificial processes or policies as sequencing of colored sectors with other related materials, location of colored sectors in classroom relative to other materials, physical and temporal accessibility of the colored sectors, whether activity with the sectors are teacher initiated or student initiated, whether students work individually or together, students' freedom of choice in selecting the colored sectors and potential collaborators for work with the colored sectors, student choice as to the kind of activity done with the sectors.

Finally, any design-test-verify process for colored sectors is also an artifact. Montessori often based her materials on manipulatives designed by other pedagogues (such as Froebel and Seguin) and refined these designs through experimental and observational methods that she took from anthropology and experimental psychology. She also had her own criteria for verifying the efficacy or determining the quality of materials, including the ease with which children could handle the materials and the intensity with which children focused on work with the materials.

Many details of Montessori's design-test-verify process are not known. However, there is a great amount of informal knowledge and experience within the Montessori community regarding the design and evaluation of materials. There is also interest in this process in the wider educational research community. In 1991, Allan Collins proposed a design science of education (Collins, 1991). Since then, a growing number of researchers have engaged in design-based research and are now striving to better define the field (The Design-Based Research Collective, 2002).

As of June, 2004, I have used the colored sectors in two different classrooms with gifted children aged K-3. With one exception, they had no trouble ordering or comparing the sectors. Students in both classes made significant gains in tests of angle understanding. However, this was not the only angle activity used in either case. Further, it is evident from Montessori folklore and present practices that we must consider the child's focus or attention during activity, reflection on activity, frequency of use of material, duration of use, and long term effects in evaluating the role of materials in supporting quality learning experiences.

Further exploration of Montessori and design-based research practices and more in-class experimentation and observation are required before any robust claims can be made about the effectiveness of colored sectors. My hope is that I have demonstrated how the application of design patterns can yield designs of materials that are at least promising. The colored sectors material was designed from a single explicit design pattern (and more implicit ones). A rich set of design patterns (or even better, a complete Montessori pattern language) could serve material designers, teachers and researchers in the same way that design patterns support design work in other fields such as architecture and software design.

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The Author

TJ Leone owns and operates Leone Learning Systems, Inc., a private corporation that offers tutoring and educational software. He has a BA in Math and an MS in Computer Science, both from the City College of New York. He spent two years in graduate studies in education and computer science at Northwestern University, and six years developing educational software there. He is a former Montessori teacher and currently teaches gifted children on a part time basis at the Center for Talent Development at Northwestern University in addition to his tutoring and software development work. His web site is http://www.leonelearningsystems.com.