Fraction Circles and Polygons: Understanding External Angles

Ages

Five and up.

Prerequisites

This activity should be done after the child has experience matching congruent polygons. In a Montessori classroom, matching polygons to insets in the geometric cabinet is sufficient.

Materials

Fraction circles and polygons. If standard Montessori versions of this material are not available, fraction circles may be cut from cardboard or plastic. Polygons may also be cutout or printed on paper or cardboard. Note that standard Montessori fraction circles are 10 cm in diameter and standard Montessori polygons are 10 cm in height.

Presentation (Individual or small group)

Invite the child(ren).
Begin arranging the fraction circles and polygons as shown (Figure 1).

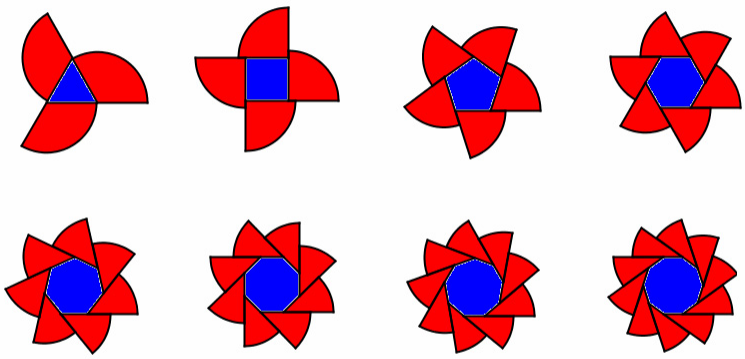

Figure 1. Polygons and the sectors of a fraction circle can be arranged to show that the external angles of regular polygons add up to 360 degrees.

Invite the child(ren) to help you complete the arrangement or leave them to complete the arrangement on their own.

Variations

After the child has experience with this activity, she may be asked to match a given fraction circle to the corresponding polygon. If the child has difficulty with this exercise, try again at a later time with only the first four polygons and the 3-, 4-, 5- and 6-sector fraction circles.

Extensions

After the polygons and fraction circles have been arranged, take out a small model of a turtle. Show the child how the turtle can "walk" around the edges of a polygon and "turn" at the corners (Figure 2). When the turtle reaches a corner, show how he turns through the angle of the sector until he faces the other radius of the sector.

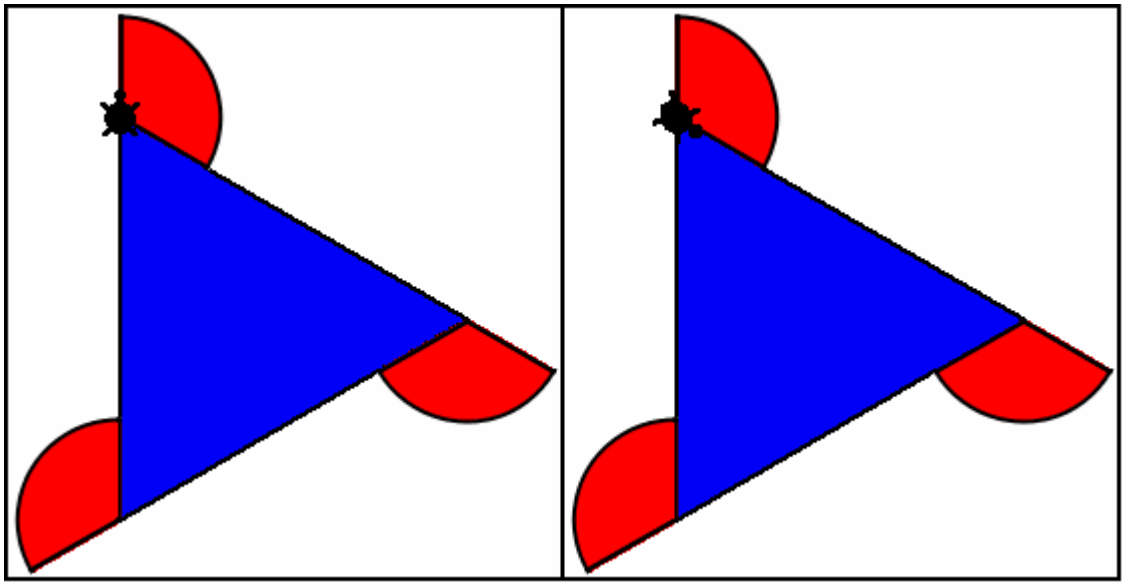

Figure 2. To trace the corner of an equilateral triangle, the turtle must rotate one third of a circle. Three hundred sixty degrees divided by three gives a turn of 120 degrees, which is the measure of the external angles of an equilateral triangle. In general, turtle turns are equal in measure to the external angles of polygons (i.e., supplementary to the internal angles), and the sum of the external angles (or turtle turns) for a polygon is always 360 degrees.

Points of Interest

Red sectors have central angles that are supplementary to the interior angles of the polygons (i.e., the central angles of the sectors are external angles of the polygons). The sectors from the three-sector fraction circle contain the external angles of the three-sided polygon, the sectors from the four-sector fraction circle contain the external angle of the

four-sided polygon, etc. After the child has experience measuring the angles of sectors, another point of interest is the fact that the external angles of the polygons all add up to 360 degrees.

Control of Error

The teacher checks to make sure that the child correctly matches fraction circles and polygons and that sectors show external angles of polygons.

Direct Aims

Develop analysis of polygons in terms of number of sides and angles.

Indirect Aims

Introduce idea of supplementary angles and the idea of internal and external angles. Introduce idea that the sum of the external angles of a regular polygon add up to 360 degrees, with each external angle of an n-gon equal to 360/n. Preparation for drawing polygons in Logo. Introduce Total Trip Theorem of turtle geometry as it relates to polygons: "If a turtle takes a trip around the boundary of [a polygon] and ends up in the state in which it started, then the sum of all turns will be 360 degrees" (Papert, Mindstorms, 1993, p. 76).