Application 5.5#1

According to the rule given on page 139 of the text, the *n*th term of the arithmetic sequence is $A(n) = a + (n-1)d$. Let's make up a function *S* for the arithmetic series, where $S(n) = \sum_{k=1}^{n} a + (k-1)d$. The claim on page 147 is that this sum *S*(*n*) is equal to $\frac{n(2a+(n-1)d)}{2}$. We want to show that this is true for any *n*.

1. First, we show that it is true when *n* = 1:

   $S(1) = a + (1-1)d = a + 0d = a + 0 = a,$ because S(1) is just the first term in the arithmetic sequence. Also, we have
   $\frac{1(2a+(1-1)d)}{2} = \frac{1(2a+0d)}{2} = \frac{1(2a+0)}{2} = \frac{1(2a)}{2} = \frac{2a}{2} = a.$
   This shows that $S(1) = \frac{n(2a+(n-1)d)}{2}$ when *n*=1.

2. Assume that the rule works for *n*=*k*. Show that it must work for *k*+1. If it holds for *k*,

$$S(k) = \frac{k(2a+(k-1)d)}{2}.$$

   If we add the (*k*+1)th term in the arithmetic sequence to both sides, the left side will be $S(k) + a + kd$, which is equivalent to *S*(*k*+1), because the definition of an arithmetic series tells us that the *n*th term is obtained by adding $a + (n-1)d$. The (*k*+1)th term in the arithmetic sequence is $a + [(k-1)+1]d = a + kd$. For the proof to work, the right side will need to represent the formula for *S*(*k*+1). Adding $a + kd$ to both sides, we have

$$S(k) + a + kd = \frac{k(2a+(k-1)d)}{2} + a + kd,$$

$$S(k+1) = \frac{k(2a+(k-1)d)}{2} + a + kd = \frac{k(2a+(k-1)d)}{2} + \frac{2a}{2} + \frac{2kd}{2}$$
$$= \frac{2ak + k(k-1)d + 2a + 2kd}{2} = \frac{2ak + 2a(1) + (k-1)kd + 2kd}{2}$$
$$= \frac{2a(k+1) + kd(k-1+2)}{2} = \frac{(k+1)(2a) + (k+1)kd}{2} = \frac{(k+1)(2a+kd)}{2}.$$

   So the rule is true for *k*+1, which means it holds for all *n*.

Application 5.5#2

To solve this problem, I show that the average of the first and last terms multiplied by *n* is just the formula for the arithmetic series:

$$\frac{n(2a+(n-1)d)}{2}$$

1. First, we show that it is true when $n = 1$. By definition, the first term in the arithmetic sequence is *a*, so, when there is only one term, the average of the first and last terms multiplied by the number of terms is:

$$\frac{1(a+a)}{2}=\frac{1(2a)}{2}=\frac{1(2a+0d)}{2}=\frac{1(2a+(1-1)d}{2},$$

   which is the formula for the arithmetic series when n=1. To derive this, I remembered from Application 5.5#1 that

$$\frac{1(2a+(1-1)d)}{2}=\frac{1(2a+0d)}{2}=\frac{1(2a+0)}{2}=\frac{1(2a)}{2}=\frac{2a}{2}=a,$$

   and just kind of worked backwards.

2. Assume that the rule works for n=k. Show that it must work for k+1. If it holds for *k*, then the value of an arithmetic series with *k* terms is:

$$k\frac{(a+a+(k-1)d)}{2}=\frac{k(2a+(k-1)d)}{2}.$$

   The (k+1)th term in a sequence is

$$a+kd,$$

   and the arithmetic formula for a series of k+1 terms is

$$\frac{(k+1)(2a+kd)}{2},$$

   so we need to show that

$$\frac{k(2a+(k-1)d)}{2}+a+kd=\frac{(k+1)(2a+kd)}{2},$$

   which we already calculated in Application 5.5#1.