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Mathematical Problem-Solving with Logo – Part 2

Continuing work on Application 1.3

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November 2004

Introduction

This guide accompanies the text *Approaching Precalculus Mathematics Discretely* by Philip G. Lewis.

In this unit, we continue a discussion of Application 1.3 on pages 11-14. There are different ways to approach this problem mathematically. Here we seek to clarify the particular approach taken by Lewis, and discuss strategies for problem-solving with Logo.

Where were we?

In the section called "Understanding what is required" in Part 1, we enumerated four steps to solving the problem:

1. Plot two points, *A* (the house) and *B* (the barn).
2. Reflect *B* over the *x* axis.
3. Draw a line from *A* to the reflected image of *B* (B').
4. Determine the point P at which the line intersects the *x* axis.

In part 1, we looked at ways to randomly choose two plot points and we looked at the procedure `xreflect` to reflect B over the *x* axis. In Part 2, we will see how to find the intersection of $\overline{AB'}$ and the *x* axis.

Drawing a figure

Figures are useful ways of representing relationships between different pieces of given data and the unknown (Polya, 2004). In Application 1.3, a figure is drawn for you on page 11. In the figure, known values for points A, B and B' are shown. The unknowns include the distance between A and B' and the point P.

This is a powerful figure. Look at what it gives us. It clearly shows that $\triangle AQB'$ is a right triangle with hypotenuse $\overline{AB'}$. This calls to mind the Pythagorean theorem, which tells us that $(AQ)^2 + (QB')^2 = (AB')^2$. We can use this information to work out our `distance` procedure. It is also clear from the figure that $\triangle AMP \sim \triangle AQB'$. So we know that $\frac{AM}{AQ} = \frac{MP}{QB'}$. Finding the distance MP can help us find the point P.

The figure also gives us specific coordinates. Points A, B and B' are clearly marked. Since A is at (-30, 50), we know that M is at (-30, 0). Since A is at (-30, 50) and B' is at (150, -100), we know that Q is at (-30, -100).

Knowing those coordinates also makes certain distances obvious. The distance between A and M is 50. The distance between A and Q is 50 + 100 = 150. The distance between Q and B' is 30 + 150 = 180.

That's a lot of information in one figure, and I was only focusing on information relevant to our problem!

What if we aren't given a figure for a problem? How do you draw a good figure? One important strategy is to draw a figure of *what you would see if the problem were already solved*.

In the case of Application 1.3, P is unknown, but we put it into the figure anyway, with as much information as we have. For example, we know that P is at the intersection of $A\,B'$ and the x axis, so we draw it accordingly, even though we can't label it with exact coordinates. The figure also *shows the known* coordinates of A, B and B'.

Also, the figure *highlights relationships between the known information and the unknown*. The two key relationships that are illustrated in this figure are the reflection of B over the x axis and similarity of $\triangle AMP$ and $\triangle AQB'$.

Sometimes the right figure for at problem is obvious. Often, the figure develops as you work through the problem. It is generally helpful to *fill in the figure with data as you acquire it*. For example, as you calculate the lengths of $\overline{AM}$, $\overline{AQ}$, and $\overline{QB'}$, and the coordinates of various points, you should add these to your figure.

Drawing with Logo

In addition to drawing figures with pencil and paper, it is useful to draw with Logo as you work through a problem.

Decide what lines in the figure you'd like to draw with Logo. You can draw dashed lines with this procedure:

```
to dash :length
show (se [pos=] pos)
pd
repeat int (:length / 8) [pd fd 4 pu fd 4]
end
```

If you want to draw circles around any particular points, you can use these procedures:

```
to circlepoint :point :radius
do.circlepoint :point :radius pos heading
end

to circler :radius
repeat 360 [fd (pi * :radius) / 180 rt 1]
end

to do.circlepoint :point :radius :startpos :startheading
pu
setpos :point
fd :radius
rt 90
pd
circler :radius
pu
setpos :startpos
seth :startheading
end
```

The Author

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Reference

Polya, G. (2004). *How to Solve It* (Expanded Princeton Science Library Edition ed.). Princeton and Oxford: Princeton University Press.